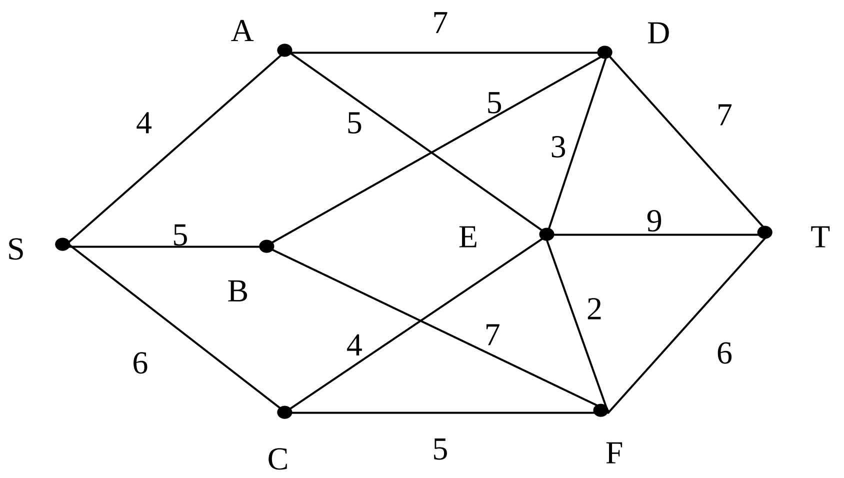
Minimum Connector Problems

A cable TV company is installing a system of cables to connect all the communities in a region. The numbers in the following diagram show the distances between the communities.



Use trial and improvement to solve the following tasks.

(a) Find a layout that will connect all the towns in the region.
(b) What is the amount of cable needed?
(c) Find a layout that uses the least amount of cable.

Primm’s Algorithm for Minimum Connector Problems

Step 1: Choose a starting vertex

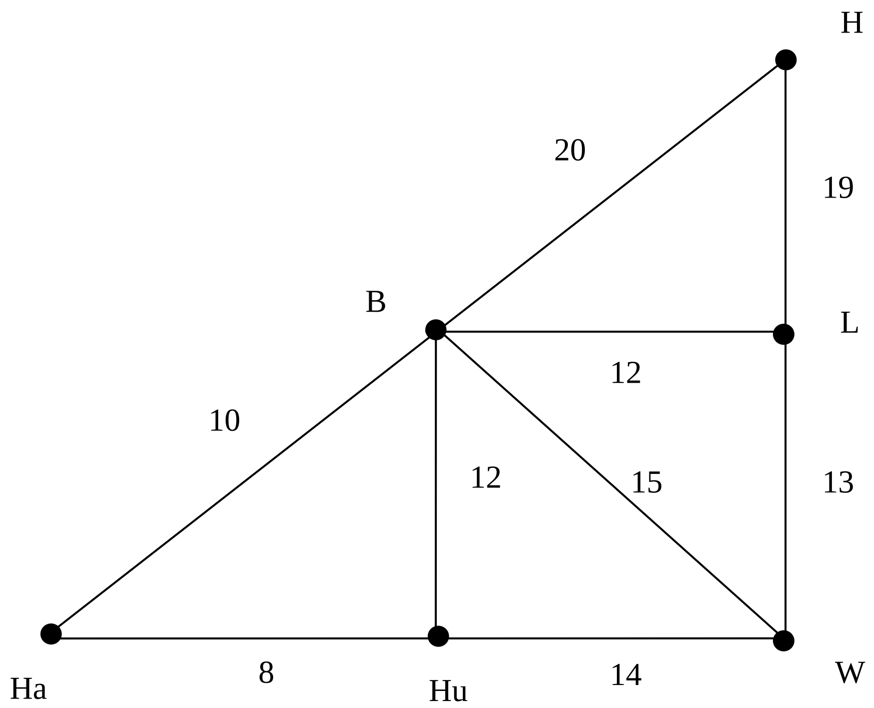
Step 2: Join this vertex to the nearest vertex into the
... existing system.

Step 3: Join the next nearest vertex not already in the
... solution, to any vertex in the solution, provided it
...does not form a cycle.

Step 4: Repeat until all vertices have been included.

A Worked Example using Prim's Algorithm

A cable TV company is installing a system of cables to connect all the towns in a region. The numbers in the following diagram show the distances in miles between the towns.

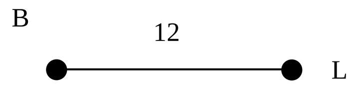


- Find a layout that will connect all the towns in the region.
- Find a different layout which uses less cable.
- What is the least amount needed?

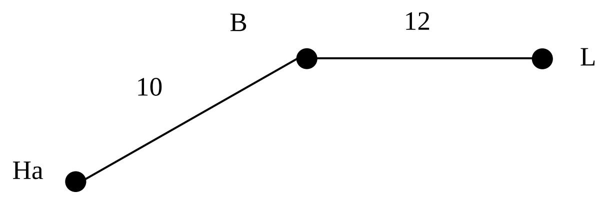
Solution

Choose a starting vertex, say L

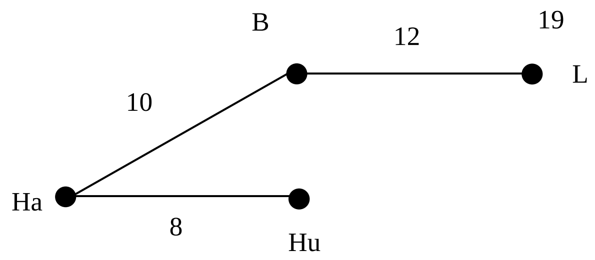
LB is the smallest edge joining L to the other vertices. Put LB into the solution.



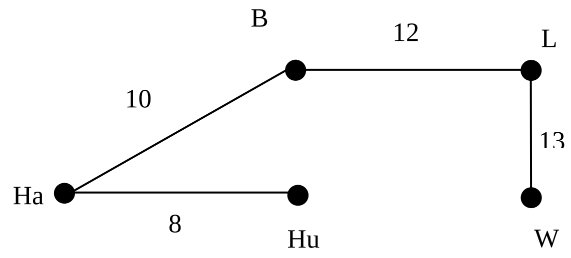
BHa is the smallest edge joining L and B to the other vertices. Put edge BHa into the solution.



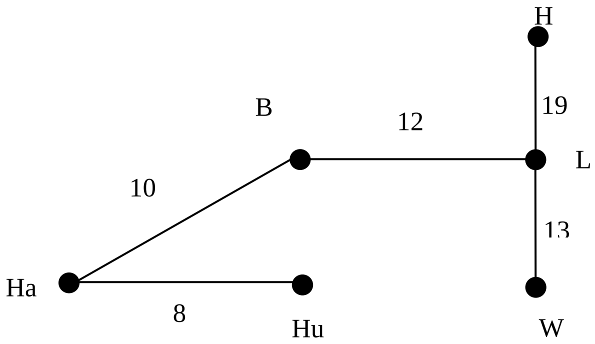
HAHu is the smallest edge joining L, B and Ha into the solution. Put HaHu into the solution.



LW is the smallest edge joining L, B, Ha and Hu to the other vertices. Put LW into the solution.



LH is the smallest edge joining L, B, Ha, Hu and W to the other vertices. Put LH into the solution.



We have now connected all the vertices into the spanning tree.

The least length of cable needed is 62 miles.

Kruskal’s Algorithm for Minimum Connector Problems

Step 1: Rank the edges in order of length

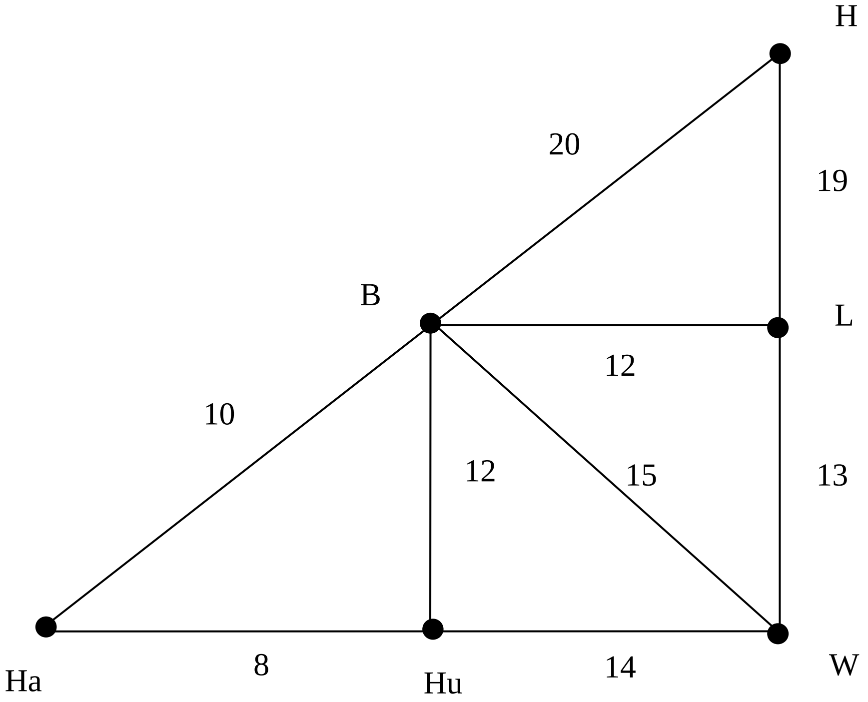
Step 2: Select the shortest edge in the network.

Step 3: Select, from the edges which are not in the
... solution, the shortest edge which does not form a
...cycle. (Where two edges have the same weight –
...select at random.)

Step 4: Repeat step 3 until all the vertices are in the
... solution.

A Worked Example using Kruskal's Algorithm

A cable TV company is installing a system of cables to connect all the towns in a region. The numbers in the following diagram show the distances in miles between the towns.



- Find a layout that will connect all the towns in the region.
- Find a different layout which uses less cable.
- What is the least amount needed?

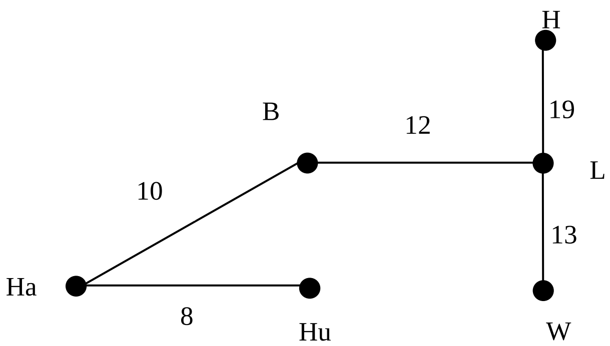
Solution

First, rank the edges in order of length

1. Ha-Hu 8
2. Ha-B 10
3. B-L 12
4. B-Hu 12
5. L-W 13
6. Hu-W 14
7. B-W 15
8. L-Ha 19
9. B-Ha 20

Order of connection into the solution:

	Ha-Hu	8	
	1. Ha-B	10	
	2. B-L	12	These two edges have the same length, the order is arbitrary
This edge forms a loop	....B-Hu	12	
	3. L-W	13	
This edge forms a loop	....Hu-W	14	
This edge forms a loop	....B-W	15	
	4. L-Ha	19	
This edge forms a loop	....B-Ha	20	



The least length of cable needed is 62 miles.

Gallery Walk for Primm and Kruskal’s Algorithms

1. Home Group Task
   .. Shortest Path Problem by trial and error

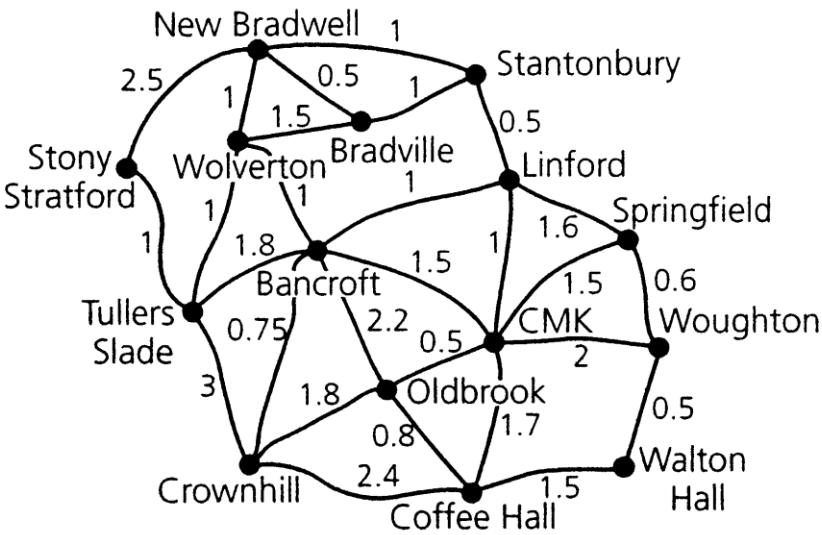
2. Working Group Activity
   .. problem using the algorithm
   .. group A uses Primm
   .. group B uses Kruskal

3. Poster walk and discussion

4. Home Group Task
   .. compare the efficiency of each algorithm; which might be easier to write software for?

Further Exercise

The communities in the city of Milton Keynes are to be connected to a new drainage system. Each community is to be connected to the sewer, positioned in CMK, either directly or via another community.



(a) Use Prim's algorithm and Kruskal's algorithm to find a layout that uses the least amount of sewage pipe.

(b) Compare the efficiency of each algorithm.